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Report on the PhD Thesis

"Differential Variational Inequalities Related to Primal-dual Projection Approximation Schemes for Operator Inclusions"

submitted by
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The PhD thesis of Krzysztof Rutkowski focusses on continuous and discrete time best approximation approaches for inclusion problems governed by maximally monotone operators. These are original and novel approaches which are fundamental for the development of solution methods for such monotone inclusion problems. These problems have been in the last decade in the focus of the optimization community, since maximally monotone inclusions can be seen as general formulations of the systems of KKT/optimality conditions for nonsmooth convex optimization problems. Nonsmooth convex optimization problems continue to play an important role in real-life applications in signal and image processing, clustering, risk analysis, machine learning, etc.
Continuous time approaches to optimization problems and monotone inclusions are unifying schemes which lead via temporal discretization to various numerical algorithms. Their asymptotic analysis usually relies on the construction of suitable energy functions and the study of their dissipation properties, and usually provide new ideas and techniques that can be translated to the convergence analysis and the convergence rates analysis of the corresponding iterative schemes. The dynamical system proposed by Mr. Rutkowski in his PhD thesis has the particularity that it is governed by a vector field which is not globally Lipschitz continuous. However, the candidate has succeeded to show, by relying on Lipschitz-like properties of moving polyhedral sets, that this vector field is locally Lipschitz continuous, which has been the crucial argument for proving the existence and uniqueness of solutions. This beautiful and not at all elementary result is the most important contribution of his PhD thesis.

The PhD thesis of Krzysztof Rutkowski has 118 pages and is structured in six chapters besides the introductory chapter and the bibliography. It relies on 4 research articles of the PhD candidate, all published in very good journals in optimization, applied mathematics and

differential equations, whereas one of these works has appeared in SIAM Journal on Optimization, which is the flagship journals of the optimization community.

Structure and contents of the PhD thesis

The introduction describes the main results of the paper and the organization of the thesis, which consists in a review of the most important results of each chapter.

Chapter 1 introduces some elements of operator theory, convex analysis and the theory of maximally monotone operators. In particular, the notion of cone is revisited in the first section of this chapter, whereas in the second one different definitions and properties related to nonexpansive operators are recalled. A particular attention is paid to the notion of resolvent of a maximally monotone operator and to the one of proximal operator of a proper, convex and lower semicontinuous function, which is the resolvent of its convex subdifferential.

In the second chapter of the thesis composite monotone inclusion problems are introduced along with their Attouch-Thera dual formulations and the related Kuhn-Tucker sets. The same is done for systems of monotone inclusions, the investigation of which is justified through the fact that they result as systems of optimality conditions for structured convex optimization problems.

Chapter 3 is the first of the thesis which contains consistent mathematical results. It starts with a revisit of the Fejer approximation algorithm for approaching a nonempty convex and closed subset of a real Hilbert space and of its corresponding convergence result. The Fejer approximation algorithms is then applied to the problem of approaching an element of the Kuhn-Tucker set of a monotone inclusion problem involving the sum of two maximally monotone operators, and also of a composite monotone inclusion problem. The subsequent section is dedicated to the Haugazeau approximation algorithm for approaching an element of a nonempty convex and closed set which, different from the Fejer approximation algorithm that exhibits only weak convergence, generates a sequence that converges strongly to the projection of the starting point onto the set. An algorithm of forward-backward-type from the literature, designed to find a zero in the sum of a single-valued α-ism and a set-valued maximally monotone operator, which uses the Haugazeau approximation algorithm to approach the corresponding Kuhn-Tucker set and inertial steps, is revisited. A relaxed version of this inertial forward-backward method is introduced in the subsequent section, also for composite monotone inclusions, relying on a generic relaxed Haugazeau best approximation scheme, which is investigated in the general setting and also proved to preserve the strong convergence properties.

In Chapter 4, a dynamical system related to the general relaxed Haugazeau approximation algorithm is formulated and investigated, first from the point of view of the existence and uniqueness of the generated trajectory. In a first instance, the existence and uniqueness of solutions on finite intervals is shown, followed by the proof of the extendibility of these solutions to the whole half-line. In the last section of the chapter, the asymptotic behaviour of the solutions of the dynamical system is studied.

The focus of Chapter 5 is on projected dynamical systems, for which first the uniqueness and

existence of solutions is addressed. The solutions are related to the ones of more general differential variational inequalities. The dynamical system investigated in Chapter 4 is shown to correspond to a particular projected dynamical system. In addition, projected dynamical systems associated with the relaxed Haugazeau scheme, the forward-backward algorithm for monotone inclusions, and the relaxed primal-dual best approximation algorithm are proposed and investigated from the point of view of the existence and uniqueness of solutions.

The investigations in Chapter 6 are by far the most consistent and innovative ones. They address the Lipschitz-likeness of moving polyhedral sets for which normal vectors and right-hand sides are locally Lipschitz continuous functions and the local Lipschitzness of projections of a given point onto such moving polyhedral sets. The latter are related to the local Lipschitzness of the right-hand side of the dynamical system studied in Chapter 4. In the analysis, basic constraint qualification conditions related to multifunctions and sets play an important role. Examples illustrating the Lipschitzness of projections on moving polyhedral sets under different constraint qualifications are provided.

Assessment of the thesis

The thesis of Mr. Krzysztof Rutkowski is an excellent piece of research work, which fulfils all the expectations one has from a PhD thesis in mathematics. The main original contribution of this work is to be seen in the fact that the candidate proposes strongly convergent numerical algorithms of Haugazeau type for approaching the corresponding Kuhn-Tucker set, a dynamic approach to this discrete one, and also studies the local Lipschitzness of projections onto moving polyhedral sets.

The PhD thesis proves that the candidate has a broad knowledge and understanding in different areas of mathematics, like optimization, maximal monotone operator theory, convex analysis, nonlinear analysis, and numerical analysis. His scientific profile has a very solid theoretical component. I strongly recommend the excellent PhD thesis of Mr. Krzysztof Rutkowski for acceptance.

Yours sincerely,

Radu Boţ